Origin of Mass and Scattering Amplitudes: from Higgs to Pauli, Kaluza-Klein and Chern-Simons

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Wuhan, December 15, 2025

References

- **Based on series of works:**

You, Song, HJH, Han, PRD(2025), 2412.16033
Han, HJH, You, Song, PRD-Lett (2025), 2412.21045

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H.X. Liu, Z.X. Yi, HJH, arXiv:2512.10870 [hep-th] (66 pp).
Y.F. Hang, HJH, C. Shen, JHEP (2022), 2110.05399 [hep-th].
Y.F. Hang, HJH, C. Shen, Research (2023), 2406.13671 [hep-th].

- **Early works:**

Chivukula, Dicus, HJH, PLB (2002) [hep-ph/0111016]
Chivukula, HJH, PLB (2002) [hep-ph/0201164]
H.J. He, IJMP (2005), APS-2004 [hep-ph/0412113]
Chivukula, Dicus, HJH, Nandi, PLB (2003) [hep-ph /0302263]

Making of the Standard Model (123)

$$
\begin{aligned}
\mathcal{L} = &\\
&-\frac{1}{4g'^4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4g^2}W^a_{\mu\nu}W^{\mu\nu a} - \frac{1}{4g_s^2}G^a_{\mu\nu}G^{\mu\nu a}\\
&+\bar{Q}_i i\not{D}Q_i + \bar{u}_i i\not{D}u_i + \bar{d}_i i\not{D}d_i + \bar{L}_i i\not{D}L_i + \bar{\ell}_i i\not{D}\ell_i\\
&+\left(Y_u^{ij}\bar{Q}_i u_j \tilde{H} + Y_d^{ij}\bar{Q}_i d_j H + Y_l^{ij}\bar{L}_i \ell_j H + c.c.\right)\\
&-\lambda(H^\dagger H)^2 + \lambda v^2 H^\dagger H - (D^\mu H)^\dagger D_\mu H
\end{aligned}
$$

Missing ν_R

No DM

No Dark Energy

Is SM Structure complete? → not really !!

→ **What is really Missing in the SM ???**

1) **ν_R Within SM Structure not yet found !**

2) **No Dark Matter (? X)**

3) **No Dark Energy (? √)**

Making of the Standard Model (123)

$$\mathcal{L} =$$
$$-\frac{1}{4g'^4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4g^2}W^a_{\mu\nu}W^{\mu\nu a} - \frac{1}{4g_s^2}G^a_{\mu\nu}G^{\mu\nu a}$$
$$+\bar{Q}_i i\not{D}Q_i + \bar{u}_i i\not{D}u_i + \bar{d}_i i\not{D}d_i + \bar{L}_i i\not{D}L_i + \bar{\ell}_i i\not{D}\ell_i$$
$$+\left(Y_u^{ij}\bar{Q}_i u_j\tilde{H} + Y_d^{ij}\bar{Q}_i d_j H + Y_l^{ij}\bar{L}_i\ell_j H + c.c.\right)$$
$$-\lambda(H^\dagger H)^2 + \lambda v^2 H^\dagger H + (D^\mu H)^\dagger D_\mu H$$

Missing ν_R

No Gravity!

SM Structure is NOT yet complete ....

→ What is *Missing* in the SM ??

→→ No full understanding on Quantum Gravity at both the Largest and Smallest Scales !!

→→ UV-IR correspondence ? …

Origin of Neutrino Mass Generation
and How to Probe it ?

You, Song, HJH, Han, PRD(2025), 2412.16033
Han, HJH, You, Song, PRD-Lett (2025), 2412.21045

Wolfgang Pauli: Father of the Neutrinos

W. Pauli (1900-1958)

- **1914: James Chadwick discovered that β decay has continuous electron energy spectrum and could not explain it with 2-body final state.**

- **In 1929, Niels Bohr attempted to explain this by giving up energy conservation (X).**

- **1930: Pauli postulated that the final state is 3-body, containging a new massless particle with electric charge 0 & spin-1/2. This naturally explains the continuous E-spectrum + conserves energy. (√)**

- **1932: Fermi named this particle as "Neutrino".**
  1933: Fermi wrote down 4-Fermion Interaction to describe β decays in weak interaction.

E. Fermi (1901-1954)

Last Missing Piece of the SM

- For conventional SM setup before 1998, neutrinos were assumed for simplicity to be massless and have only left-handed components because the SM is structured to have all the right-handed fermions be weak singlets in each fermion family, where the right-handed neutrinos (ν_R) are pure gauge singlets and their absence does not affect the gauge anomaly cancellation of the SM.

- Weinberg realized that without ν_R, the left-handed neutrinos can acquire small Majorana masses from a gauge-invariant dimension-5 operator (LLHH) that is suppressed by a large UV cutoff scale $\Lambda_\nu \sim v^2/m_\nu$, far beyond the weak scale.

- However, this dimension-5 operator is nonrenormalizable and its Minimal UV Completion is given by the canonical seesaw with $\Lambda_\nu = M_R$ after adding back ν_R for each fermion family.

- The existence of ν_R is predicted by the SM structure and provides the minimal UV completion for dimension-5 Weinberg operator through canonical Seesaw Mechanism, naturally generating light neutrino masses.
- Yet, ν_R points to a brand-new seesaw scale $\Lambda_\nu \sim v^2/m_\nu \sim 10^{14}$GeV that is far beyond SM.

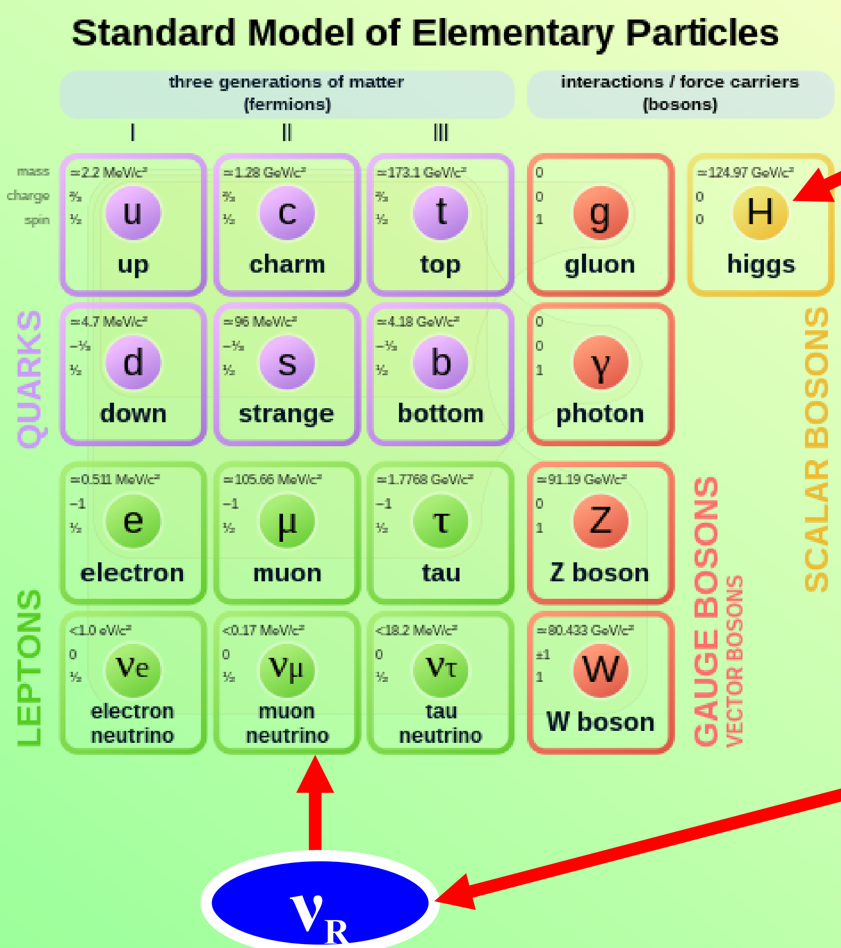
- →→ It is extremely important to probe ν_R as the Last Missing Piece of the SM and test Neutrino Mass-Generation via canonical Seesaw Mechanism.

→→ SM prediction ν_R could be wrong by EXP, but again its chance of success is high !

Structures of 2 SM

SM of Particle Physics + SM of Cosmology

（粒子物理标准模型 + 宇宙学标准模型）



Higgs h(125GeV) (2012) is actually **NOT** the Last Building Block of SM .

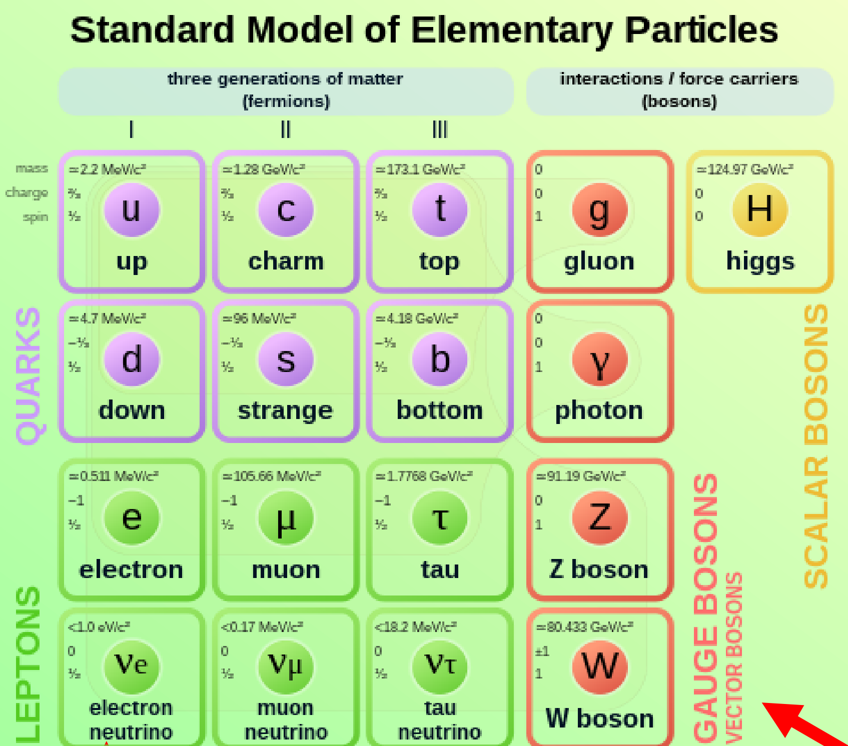
But, ν_R is the Last Missing Piece of SM !!

GR - Einstein-Hilbert Action:

$$S = \int d^4x \sqrt{-g}\,[\kappa^{-2}(R+\Lambda)+\mathcal{L}_m]$$

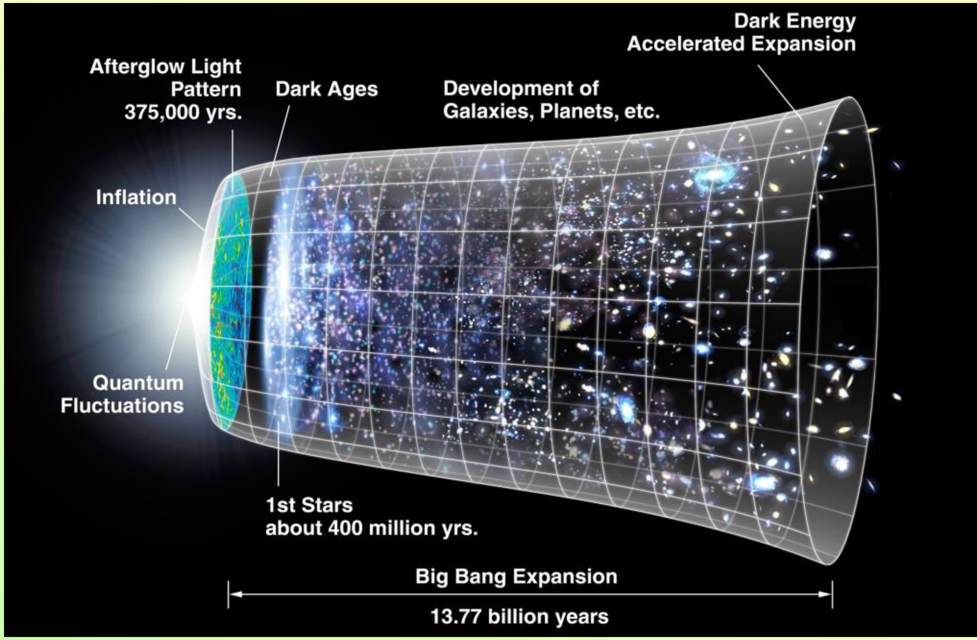
SM of Particle Physics + SM of Cosmology

粒子物理标准模型 + 宇宙学标准模型



物质的基本组成 与 相互作用

Cosmological SM: ΛCDM + Inflation



物质的起源与演化

Missing ν_R

No DM??

No Dark Energy??

Cosmological Bounds on light ν Mass Sum:

Table 26.2: Summary of $\sum m_\nu$ constraints. **PDG**

	Model	95% CL (eV)	Ref.
CMB alone			
Pl18[TT+lowE]	ΛCDM+$\sum m_\nu$	< 0.54	[24]
Pl18[TT,TE,EE+lowE]	ΛCDM+$\sum m_\nu$	< 0.26	[24]
CMB + probes of background evolution			
Pl18[TT,TE,EE+lowE] + BAO	ΛCDM+$\sum m_\nu$	< 0.13	[49]
Pl18[TT,TE,EE+lowE] + BAO	ΛCDM+$\sum m_\nu$+5 params.	< 0.515	[25]
CMB + LSS			
Pl18[TT+lowE+lensing]	ΛCDM+$\sum m_\nu$	< 0.44	[24]
Pl18[TT,TE,EE+lowE+lensing]	ΛCDM+$\sum m_\nu$	< 0.24	[24]
Pl18[TT,TE,EE+lowE]+ ACT[lensing]	ΛCDM+$\sum m_\nu$	< 0.12	[50]
CMB + probes of background evolution + LSS			
Pl18[TT,TE,EE+lowE] + BAO + RSD	ΛCDM+$\sum m_\nu$	< 0.10	[49]
Pl18[TT,TE,EE+lowE+lensing] + BAO + RSD + Shape	ΛCDM+$\sum m_\nu$	< 0.082	[51]
Pl18[TT+lowE+lensing] + BAO + Lyman-α	ΛCDM+$\sum m_\nu$	< 0.087	[52]
Pl18[TT,TE,EE+lowE] + BAO + RSD + SN + DES-Y1	ΛCDM+$\sum m_\nu$	< 0.12	[49]
Pl18[TT,TE,EE+lowE] + BAO + RSD + SN + DES-Y3	ΛCDM+$\sum m_\nu$	< 0.13	[53]

→ **Finally, the heaviest Light Neutrino Mass should be around 0.05eV (IO) — 0.06eV (NO).**

Upper Bounds on Largest Light ν Mass

$$\text{NO}: \quad \sum m_\nu = m_3 + \sqrt{m_3^2 - \Delta m_{31}^2} + \sqrt{m_3^2 - \Delta m_{31}^2 + \Delta m_{21}^2}$$

$$\text{IO}: \quad \sum m_\nu = m_2 + \sqrt{m_2^2 - \Delta m_{21}^2} + \sqrt{m_2^2 - |\Delta m_{32}|^2}\,,$$

- **Largest Light Neutrino Mass:**

→→ **Normal Ordering (NO): 0.06 eV**
→→ **Inverted Ordering (IO): 0.05 eV**

Beyond SM: Scale of Neutrino Mass Generation

- **Weinberg Operator:** $1/\Lambda_\nu$ (LLHH)

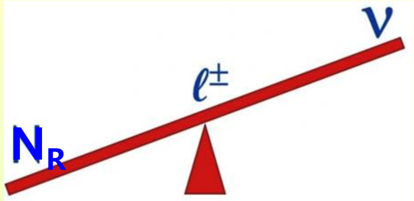
  →→ **Model-independent formulation of ν Mass Generation !**

- **Canonical Seesaw is the Minimal UV Completion.**

- **Seesaw Scale:** $\Lambda_\nu = M_R = m_D^2/m_\nu \sim 10^{14}$ GeV

  →→ Seesaw Scale is a brand-new Scale beyond SM !

Neutrino Seesaw



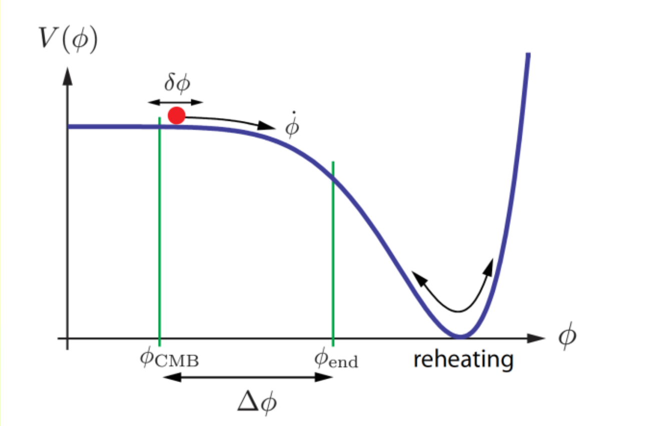
→ **Great Challenge:**
How to test the High Scale Seesaw ? ?

Minimal Model

You, Song, HJH, Han, PRD(2025), 2412.16033
Han, HJH, You, Song, PRDLett (2025), 2412.21045

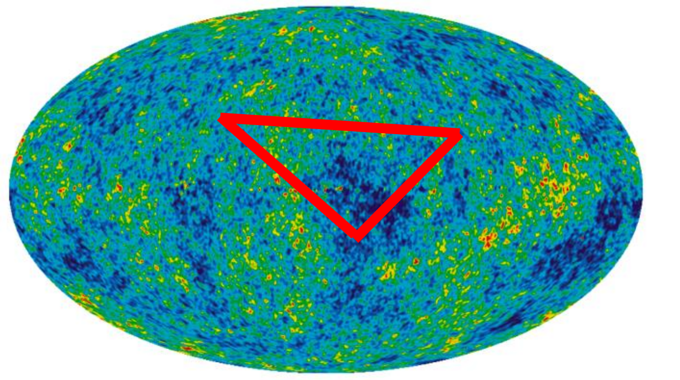
Inflation + Seesaw occur around Same Scale ~ 10^{14} GeV !

$$\Delta\mathcal{L} = \sqrt{-g}\Big[-\frac{1}{2}\partial_\mu\phi\partial^\mu - V(\phi) + \overline{N}_{\rm R} i\partial\!\!\!/ N_{\rm R} + \frac{1}{\Lambda}\partial_\mu\phi\,\overline{N}_{\rm R}\gamma^\mu\gamma^5 N_{\rm R} + \Big(-\frac{1}{2}M\overline{N^{\rm c}_{\rm R}}N_{\rm R} - y_\nu\,\bar{\ell}_{\rm L}\tilde{\mathbb{H}}N_{\rm R} + {\rm H.c.}\Big)\Big],$$



- **$V(\phi)$ is the potential for inflation, is unknown but dominated by the mass term after inflation.**
- **Derivative coupling to keep the flatness of the inflaton potential (shift-symmetry).**
- **$\Lambda > 60\,H_{\rm inf}$ to keep perturbative unitarity.**
- **Inflaton coupling with SM fermions does not affect the analysis.**
- **After inflation, inflaton oscillates at the bottom of the potential until it decays into heavy N_R ($M_\phi > 2\,M_R$). The heavy neutrinos quickly decay into SM particles and reheat the universe.**

Non-Gaussianity



Non-Gaussianity is sensitive to New Physics !

- **New physics could induce large non-Gaussianity: multi-field inflation models, modulated reheating, curvaton scenario .....**
- **Current limit from Planck on local type f_{NL}~ O(10), future CMB observations, LiteBIRD O(1), large scale structure observations DESI O(1), 21cm tomography O(0.01-0.1)**
- **Non-Gaussianity can provide information to the new particle mass, spin, interactions: Cosmological Collider Signals**

Nima Arkani-Hamed, Juan Maldacena, arXiv:1503.08043
Xingang Chen, Yi Wang, JCAP 04 (2010) 027

Local type non-Gaussianity

- **The local type non-Gaussianity which is defined by Bardeen Potential:** $\Phi \equiv \frac{3}{5}\zeta$

$$\langle \Phi_{\mathbf{k_1}} \Phi_{\mathbf{k_2}} \Phi_{\mathbf{k_3}} \rangle'_{\rm local} = 2A^2 f_{\rm NL}^{\rm local} \left\{ \frac{1}{k_1^3 k_2^3} + \frac{1}{k_2^3 k_3^3} + \frac{1}{k_3^3 k_1^3} \right\}$$

- **In the limit $k_1 \sim k_2 >> k_3$, we find**

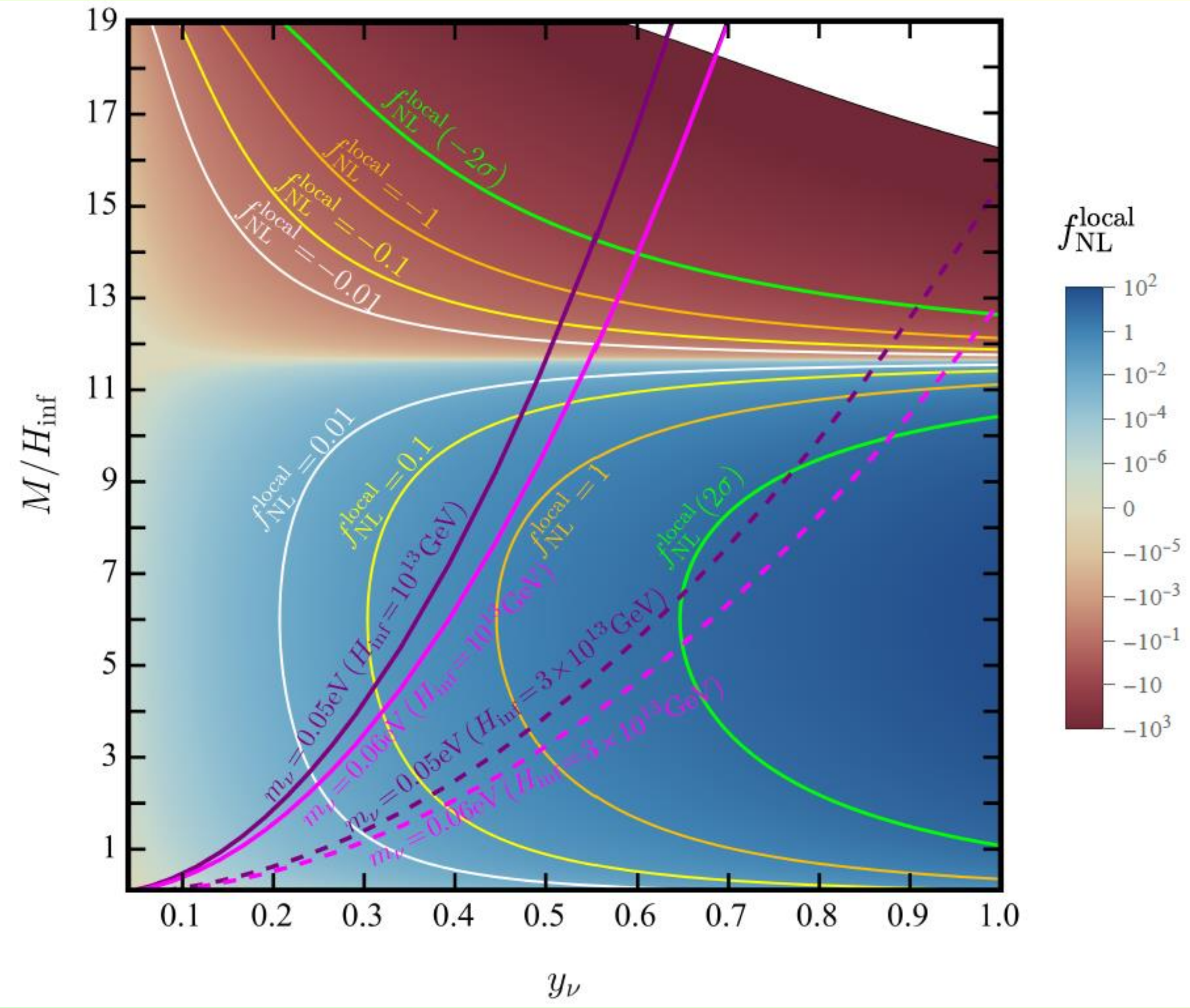
$$f_{\rm NL}^{\rm local} \sim -\frac{10}{3} \frac{z_1^3 H^3}{(2\pi)^4 \mathcal{P}_\zeta^2} \cdot \left(\frac{\lambda \bar{h}}{2H} N_e - \frac{H \cdot z_2}{4 z_1} \right)$$

$$f_{\rm NL}^{\rm local} = -0.9 \pm 5.1 \quad (68\% \text{ C.L., Planck 2018})$$

- **Benchmark for this analysis:**

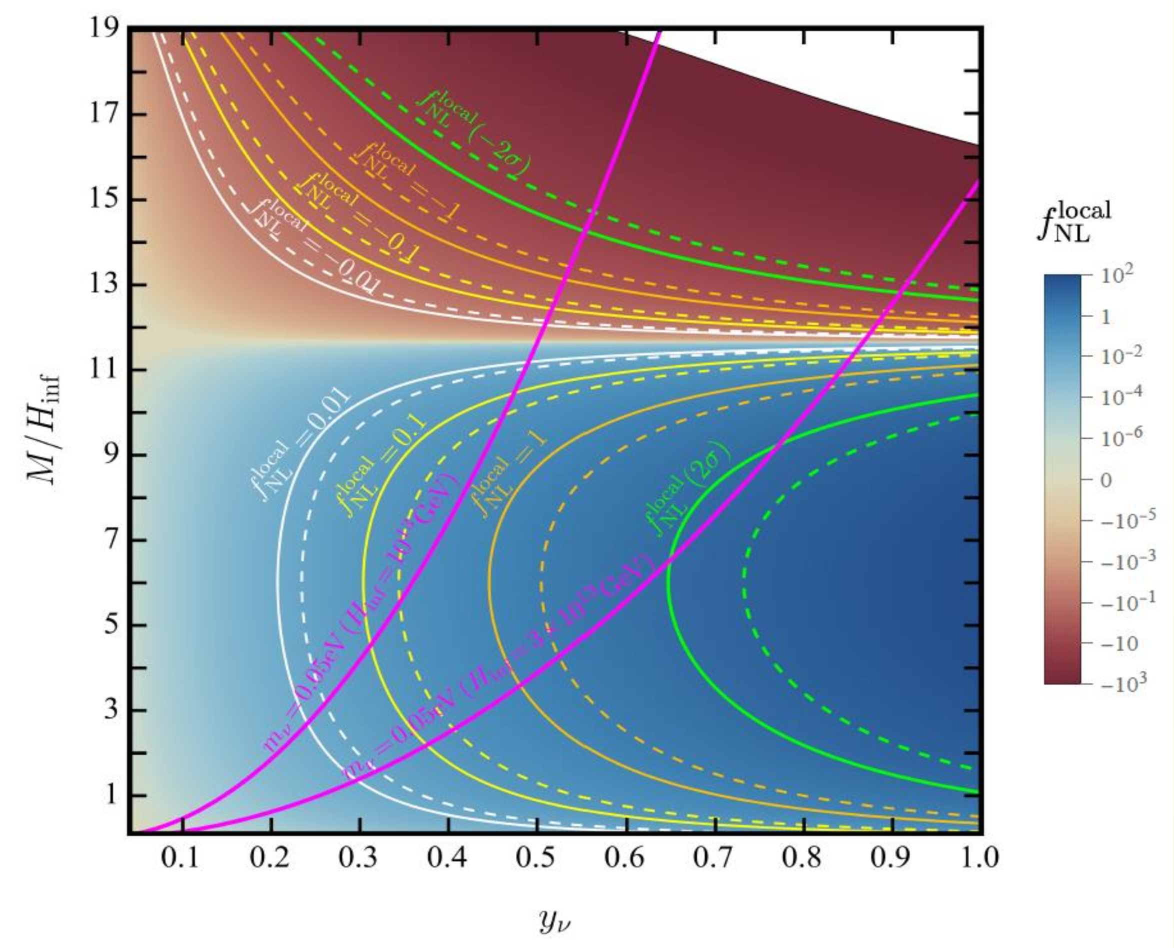
Parameters	$\mathcal{P}_\zeta$	N_e	$H_{\rm inf}$	m_ϕ	Λ	λ
Values	2.1×10^{-9}	60	$(1,3)\times10^{13}\,$GeV	$40H_{\rm inf}$	$60H_{\rm inf}$	0.01

non-Gaussianity Probes Seesaw: M_R vs Yukawa y_ν and ν_R vs ν_L Mass Scales



You, Song, HJH, Han, PRD(2025), 2412.16033
Han, HJH, You, Song, PRDLett (2025), 2412.21045

NG: Seesaw & Higgs Self-Coupling



$f_{NL}^{local} = \pm 0.01\ (\lambda = 0.01)$ — $f_{NL}^{local} = \pm 0.01\ (\lambda = 0.02)$

$f_{NL}^{local} = \pm 0.1\ (\lambda = 0.01)$ — $f_{NL}^{local} = \pm 0.1\ (\lambda = 0.02)$

$f_{NL}^{local} = \pm 1\ (\lambda = 0.01)$ — $f_{NL}^{local} = \pm 1\ (\lambda = 0.02)$

$f_{NL}^{local}(\pm 2\sigma)\ (\lambda = 0.01)$ — $f_{NL}^{local}(\pm 2\sigma)\ (\lambda = 0.02)$

$m_\nu = 0.05\text{eV}\ (H_{inf} = (1 \text{ or } 3)\times 10^{13}\text{GeV})$

You, Song, HJH, Han, PRD(2025), 2412.16033
Han, HJH, You, Song, PRDLett (2025), 2412.21045

Summary-1

- **We propose a Minimal Model incorporating inflation and seesaw through ϕ - ν_R interaction. It provides a New Realization of Higgs-modulated reheating.**

> **It gives a direct probe of Seesaw for Neutrino Mass Generation ! It links Non-Gaussianity constraint to Low Energy ν-Exps on light ν masses and to LHC measurements on Higgs self-coupling, via RG running.**

> **Seesaw generated non-Gaussianity could be probed in near future CMB or large-scale structure observations**

- **It also provides a framework of cosmological Higgs collider (particles coupling to Higgs boson could be detected)**

You, Song, HJH, Han, PRD(2025), 2412.16033
Han, HJH, You, Song, PRDLett (2025), 2412.21045

Geometric Higgs Mechanism
for KK Graviton Mass Generation

Hang and HJH, PRD(2022), 2106.04568
Research (2022), 2207.11214
Hang, Zhao, HJH, Qiu, (2024), 2406.12713

Challenges of Mass Generations in 4d

Nothing forbids Spin-0 Higgs Boson gets a huge mass from quantum corrections!
Quark/Lepton mass-scales are protected by P-violation and set by Higgs VEV through Yukawa coupling.

Fields	$m=0$		$m\neq 0$
H^0	1	=	1
$(s=0)$	$(\lambda=0)$		$(\lambda=0)$
A^μ	2	$\neq$	3
$(s=1)$	$(\lambda=\pm 1)$		$(\lambda=0,\pm 1)$
$\mathrm{h}^{\mu\nu}$	2	$\neq$	5
$(s=2)$	$(\lambda=\pm 2)$		$(\lambda=0,\pm 1,\pm 2)$
(ψ_L,ψ_R)	$1+1$	=	2
$(s=\frac{1}{2})$	$(\lambda=\frac{1}{2},-\frac{1}{2})$		$(\lambda=\pm\frac{1}{2})$

Higgs Mechanism and Beyond

- "Higgs" Mechanism can be

  1). Conventional, 2). Geometric, 3). Topological

-- Conventional Higgs Mechanism:

Mass-Generation by SSB by Vacuum Expectation Value of Higgs Boson.

-- Geomertric "Higgs" Mechanism:

Mass-Generation SSB by Compactification of Extra Dimension.

-- Topological "Higgs" Mechanism:

No SSB, construction of Gauge-Invariant Mass Term with topology,

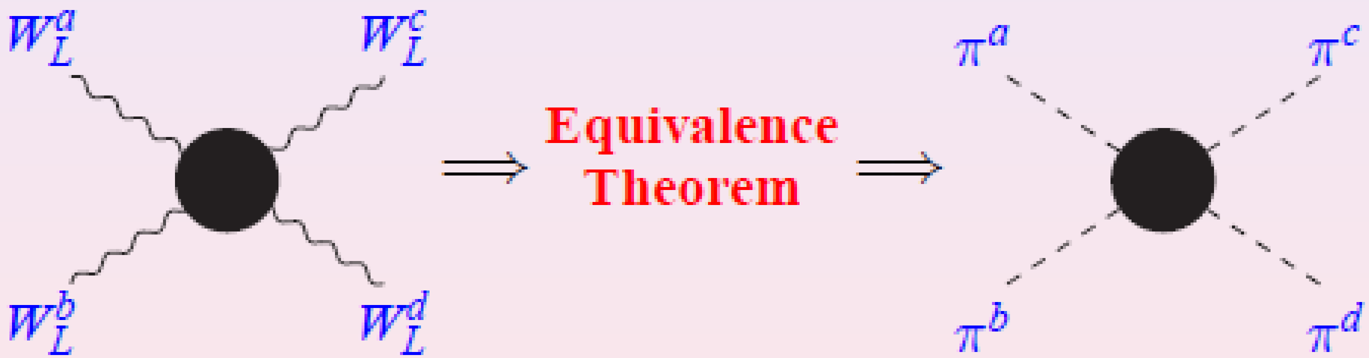
Dynamical Conversion of Physical Degrees of Freedom.

SM: ET as Bridge between Exp and EWSB

No Lose Theorem for LHC Discovery

— Equivalence Theorem —

the Bridge between Experiments and EWSB



Energy Cancellations: $E^4 \rightarrow E^0$

Unitarity Bound $\rightarrow$ $m_H < 800\text{GeV}$ or $\Lambda < 1.2\text{TeV}$

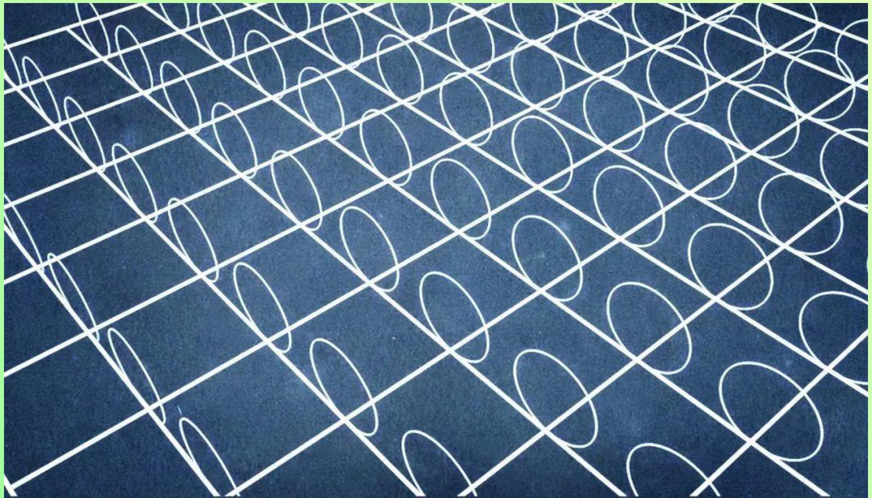
For a review: H.-J. He et al, arXiv:hep-ph/9704276.

5d: Kaluza-Klein Compactification

- We are apparently living in (3+1)d spacetime.

- But our Universe could have Extra Dimensions beyond d = 4 , except that Extra Dimensions are all curled-up !

- Simplest case is a 5^{th} dimension curled up on a circle S^1, called Kaluza-Klein (KK) Compactification (in 1920s)



Geometric "Higgs" Mechanism in 5d

- **Under compactification of flat 5d, how do KK Masses arise? ---**

▶ Consider 5d Massless Gauge Fields $\hat{A}^{aM}(x^\mu, x^5)$,

$$\begin{cases} 0 = P^2 = p_\mu p^\mu + p_5 p^5 = p^2 - p_5^2 \\ p^5 = \dfrac{n}{R}, \qquad (n = 0, 1, 2, \cdots), \end{cases}$$

(After 5d compactification)

where p^5 is quantized due to BCs, and $p^2 = p_\mu p^\mu$ is the 4-momentum-squared in 4d.

▶ Hence, in 4d we can see the "KK Tower", as a unique consequence of 5d Compactification,

$$p^2 = p_5^2 = \frac{n^2}{R^2}, \quad (n = 0, 1, 2, \cdots), \quad \Rightarrow \quad \text{KK Tower !}$$

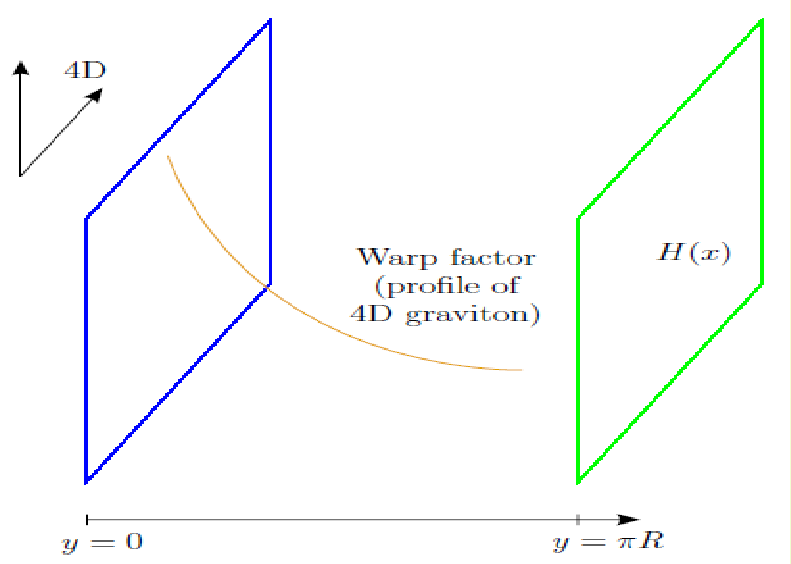
- **Masses Generation by KK Compactification:**

→ **Geometric "Higgs" Mechanism**

Chivukula, Dicus, HJH, PLB (2002) hep-ph/0111016

Warped 5d: Geometric "Higgs" Mechanism

- Warped 5d RS1 under S^1/Z_2 :



Randall-Sundrum
PRL.1999, hep-ph/9905221

- Warped 5d metric:

$$\mathrm{d}s^2 = e^{2A(y)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}y^2,$$

$$\mathrm{d}s^2 = e^{2A(z)}(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}z^2)$$

where

$A(z) = -ky,$ $\quad A(z) = -\ln(1+kz)\,,$ $\quad k = \sqrt{-\Lambda/6}$ $\quad \hat{g}_{MN} = e^{2A(z)}\hat{\eta}_{MN}$

Geometric Higgs Mechanism for KK Gravitons

- **Under S^1/Z_2 compactification, we impose the Boundary Conditions:**

$$\partial_5 \hat{h}_{\mu\nu}\big|_{x^5=0,L} = 0\,, \qquad \partial_5 \hat{\phi}\big|_{x^5=0,L} = 0\,, \qquad \hat{h}_{\mu 5}\big|_{x^5=0,L} = 0$$

- **4d Quadratic KK Lagrangian contains Mixing Terms:**

$$+\,2\mathbb{M}_n h_n \partial_\mu \mathcal{V}^\mu_n - 2\mathbb{M}_n h^{\mu\nu}_n \partial_\mu \mathcal{V}_{\nu,n} - \frac{3}{2}\mathbb{M}^2_n h_n \phi_n - 3\mathbb{M}_n \partial_\mu \mathcal{V}^\mu_n \phi_n$$

which can be eliminated by R_ξ gauge fixing:

$$\mathcal{V}^\mu_n = h^{\mu 5}_n$$
$$\phi_n = h^{55}_n$$

$$\mathcal{L}_{\rm GF} = \int_0^L \mathrm{d}z\, \hat{\mathcal{L}}_{\rm GF} = -\sum_{n=0}^{\infty} \frac{1}{\xi_n}\Big[(\mathcal{F}^\mu_n)^2 + (\mathcal{F}^5_n)^2\Big],$$

$$\mathcal{F}^\mu_n = \partial_\nu h^{\mu\nu}_n - \left(1 - \frac{1}{2\xi_n}\right)\partial^\mu h_n + \frac{1}{\sqrt{2}}\xi_n \mathbb{M}_n \mathcal{V}^\mu_n\,,$$
$$\mathcal{F}^5_n = \frac{1}{2}\mathbb{M}_n h_n - \sqrt{\frac{3}{2}}\,\xi_n \mathbb{M}_n \phi_n + \frac{1}{\sqrt{2}}\,\partial_\mu \mathcal{V}^\mu_n\,.$$

Hang, HJH, 2106.04568, 2207.11214

- **R$_\xi$ Gauge Propagators:**

$$\begin{aligned}\mathcal{D}^{\mu\nu\alpha\beta}_{nm}(p) \;=\; -\,\frac{\mathrm{i}\delta_{nm}}{2}\Bigg\{&\frac{(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}-\eta^{\mu\nu}\eta^{\alpha\beta})}{p^2+M_n^2}\\
&+\frac{1}{3}\left[\frac{1}{p^2+M_n^2}-\frac{1}{p^2+(3\xi_n-2)M_n^2}\right]\left(\eta^{\mu\nu}-\frac{2p^\mu p^\nu}{M_n^2}\right)\left(\eta^{\alpha\beta}-\frac{2p^\alpha p^\beta}{M_n^2}\right)\\
&+\frac{1}{M_n^2}\left[\frac{1}{p^2+M_n^2}-\frac{1}{p^2+\xi_n M_n^2}\right](\eta^{\mu\alpha}p^\nu p^\beta+\eta^{\mu\beta}p^\nu p^\alpha+\eta^{\nu\alpha}p^\mu p^\beta+\eta^{\nu\beta}p^\mu p^\alpha)\\
&+\frac{4p^\mu p^\nu p^\alpha p^\beta}{\xi_n M_n^4}\left(\frac{1}{p^2+\xi_n^2 M_n^2}-\frac{1}{p^2+\xi_n M_n^2}\right)\Bigg\},\qquad (2.:\end{aligned}$$

Hang, HJH, 2106.04568, 2207.11214

KK Graviton Propagator in Feynman/Unitary Gauge

- **Feynman Gauge (ξ_n=1) Propagators take very simple form:**

$$\mathcal{D}_{nm}^{\mu\nu\alpha\beta}(p) = -\frac{\mathrm{i}\delta_{nm}}{2}\frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}-\eta^{\mu\nu}\eta^{\alpha\beta}}{p^2+M_n^2}$$

- **We also derive Unitary Gauge ($\xi_n = \infty$) Propagator:**

$$\mathcal{D}_{nm,\mathrm{UG}}^{\mu\nu\alpha\beta}(p) = -\frac{\mathrm{i}\delta_{nm}}{2}\frac{\bar{\eta}^{\mu\alpha}\bar{\eta}^{\nu\beta}+\bar{\eta}^{\mu\beta}\bar{\eta}^{\nu\alpha}-\frac{2}{3}\bar{\eta}^{\mu\nu}\bar{\eta}^{\alpha\beta}}{p^2+M_n^2}$$

where $\bar{\eta}^{\mu\nu} = \eta^{\mu\nu} + p^\mu p^\nu/M_n^2$.

Hang, HJH, (2021), 2106.04568, 2207.11214

KK Graviton Propagator vs vDVZ Discontinuity

- **KK Graviton propagator in $M_n \to 0$ limit:**

$$\mathcal{D}_{nm}^{\mu\nu\alpha\beta}(p) = -\frac{\mathrm{i}\delta_{nm}}{2}\left[\frac{(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}-\eta^{\mu\nu}\eta^{\alpha\beta})}{p^2} - \frac{1-\xi_n}{p^4}(\eta^{\mu\alpha}p^\nu p^\beta+\eta^{\mu\beta}p^\nu p^\alpha+\eta^{\nu\alpha}p^\mu p^\beta \right.$$
$$\left. +\eta^{\nu\beta}p^\mu p^\alpha-2\eta^{\mu\nu}p^\alpha p^\beta-2\eta^{\alpha\beta}p^\mu p^\nu) - 4(1-\xi_n)^3\frac{p^\mu p^\nu p^\alpha p^\beta}{p^6}\right],$$
$$= -\frac{\mathrm{i}\delta_{nm}}{2}\,\frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}-\eta^{\mu\nu}\eta^{\alpha\beta}}{p^2}\,, \quad (\text{for } \xi_n=1).$$

→→ **It is free from vDVZ (van-Dam-Veltman-Zakharov) Discontinuity !**

- **Compared with conventional massless graviton propagator:**

$$\mathcal{D}_{00}^{\mu\nu\alpha\beta}(p) = -\frac{\mathrm{i}}{2}\left[\frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}-\eta^{\mu\nu}\eta^{\alpha\beta}}{p^2} + (\xi-1)\frac{\eta^{\mu\alpha}p^\nu p^\beta+\eta^{\mu\beta}p^\nu p^\alpha+\eta^{\nu\alpha}p^\mu p^\beta+\eta^{\nu\beta}p^\mu p^\alpha}{p^4}\right]$$
$$= -\frac{\mathrm{i}}{2}\,\frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}-\eta^{\mu\nu}\eta^{\alpha\beta}}{p^2}\,, \qquad (\text{for } \xi=1).$$

Hang and HJH, PRD.2022 [arXiv:2106.04568], Research.2022 [2207.11214]

KK Graviton Propagator vs vDVZ Discontinuity

- **KK Graviton propagator in $M_n \to 0$ limit:**

$$\mathcal{D}_{nm}^{\mu\nu\alpha\beta}(p) = -\frac{\mathrm{i}\delta_{nm}}{2}\left[\frac{(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}-\eta^{\mu\nu}\eta^{\alpha\beta})}{p^2} - \frac{1-\xi_n}{p^4}(\eta^{\mu\alpha}p^\nu p^\beta+\eta^{\mu\beta}p^\nu p^\alpha+\eta^{\nu\alpha}p^\mu p^\beta \right.$$
$$\left. +\eta^{\nu\beta}p^\mu p^\alpha - 2\eta^{\mu\nu}p^\alpha p^\beta - 2\eta^{\alpha\beta}p^\mu p^\nu) - 4(1-\xi_n)^3\frac{p^\mu p^\nu p^\alpha p^\beta}{p^6}\right],$$
$$= -\frac{\mathrm{i}\delta_{nm}}{2}\frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}-\eta^{\mu\nu}\eta^{\alpha\beta}}{p^2}, \quad (\text{for } \xi_n=1).$$

- **Compared with Fierz-Pauli massive graviton propagator:**

$$\mathcal{D}_{\mathrm{PF}}^{\mu\nu\alpha\beta}(p) = -\frac{\mathrm{i}}{2}\frac{\bar{\eta}^{\mu\alpha}\bar{\eta}^{\nu\beta}+\bar{\eta}^{\mu\beta}\bar{\eta}^{\nu\alpha}-\frac{2}{3}\bar{\eta}^{\mu\nu}\bar{\eta}^{\alpha\beta}}{p^2+M^2}$$

where $\bar{\eta}^{\mu\nu} = \eta^{\mu\nu} + p^\mu p^\nu / M^2$.

It has vDVZ discontinuity in $M_n \to 0$ limit!

Hang, HJH, arXiv:2106.04568, 2207.11214

KK Graviton Propagator vs vDVZ Discontinuity

- **5d KK Graviton conserves physical d.o.f in $M_n \to 0$ limit:**

$$5 = 2 + 2 + 1$$

$$\lambda = \quad 0, \pm 1, \pm 2 \qquad \pm 2 \qquad \pm 1 \qquad 0$$

where $\lambda = 0, \pm 1$ **d.o.f arise from 3 KK Goldstones** $(h_n^{\mu 5},\, h_n^{55})$ **.**

- **But, the Massive Graviton of Fierz-Pauli gravity does not conserve physical d.o.f in $M_n \to 0$ limit:**

$$5 \neq 2$$

Hang & HJH, arXiv:2106.04568, 2207.11214

Schematic Summary:
ET vs Double-Copy and from 3pt to 4pt Amplitudes

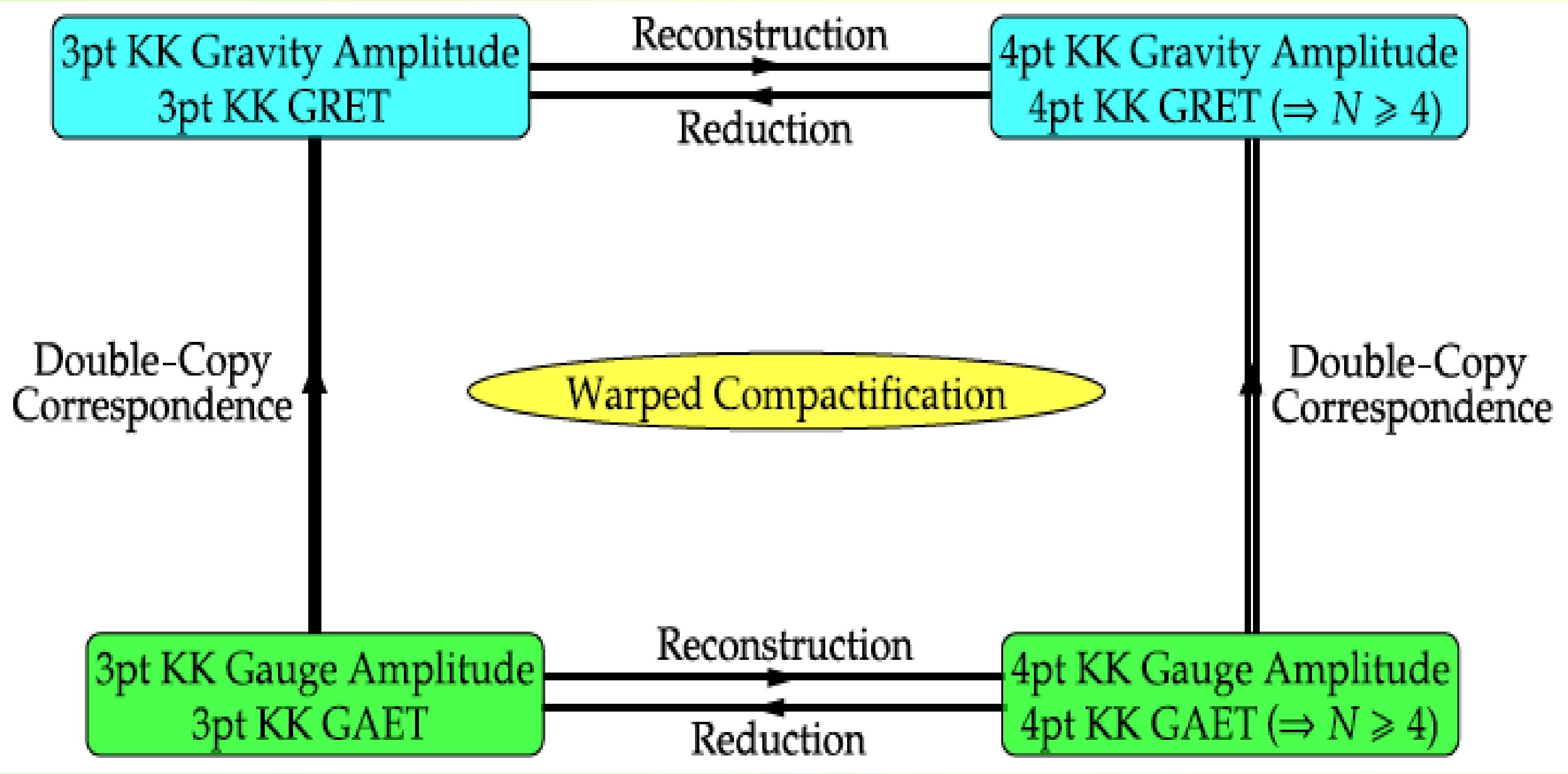
$$
\begin{array}{ccccccc}
\widetilde{\mathcal{T}}_0^{(4)}[A_5^n] & \xlongequal{\text{GAET}(4)} & \mathcal{T}_0^{(4)}[A_L^n] & \xRightarrow{\ \text{DC}_4\ } & \mathcal{M}_0^{(4)}[h_L^n] & \xlongequal{\text{GRET}(4)} & \widetilde{\mathcal{M}}_0^{(4)}[\phi_n] \\
 & \uparrow & & \uparrow & & \uparrow & \\
\widetilde{\mathcal{T}}_0^{(3)}[A_5^n] & \xlongequal{\text{GAET}(3)} & \mathcal{T}_0^{(3)}[A_L^n] & \xRightarrow{\ \text{DC}_3\ } & \mathcal{M}_0^{(3)}[h_L^n] & \xlongequal{\text{GRET}(3)} & \widetilde{\mathcal{M}}_0^{(3)}[\phi_n]
\end{array}
$$

- **This can be further extended to the case of N>4 amplitudes.**

Hang, Zhao, HJH, Qiu, JHEP(2025), 2406.12713

KK ET & Double-Copy Correspondance: 3-Point to 4-Point

- **Equivalence Theorem and Double-Copy Correspondence from 3pt KK amplitudes to 4pt KK amplitudes and from massive KK gauge amplitudes to massive KK gravitational amplitudes.**



Hang, Zhao, HJH, Qiu, JHEP(2025), 2406.12713

Geometric Higgs Mechanism via GRET

- **Gravitational Equivalence Theorem (GRET type-I):**

$$\mathcal{M}[h^L_{n_1},\cdots,h^L_{n_N},\Phi] = C^{n_j m_j}_{\rm mod}\mathcal{M}[\phi_{m_1},\cdots,\phi_{m_N},\Phi] + O(\mathbb{M}_n/E_n\text{–suppressed})$$

$$C^{n_j m_j}_{\rm mod} = C_{n_1 m_1}\cdots C_{n_N m_N} = \delta_{n_1 m_1}\cdots\delta_{n_N m_N} + O(\text{loop})$$

- **Gravitational Equivalence Theorem (GRET type-II):**

$$\mathcal{M}[h^{\pm 1}_{n_1},\cdots,h^{\pm 1}_{n_N},\Phi] = \hat{C}^{n_j m_j}_{\rm mod}\mathcal{M}[\mathcal{V}^{\pm 1}_{m_1},\cdots,\mathcal{V}^{\pm 1}_{m_N},\Phi] + O(\mathbb{M}_n/E_n\text{–suppressed})$$

$$\hat{C}^{n_j m_j}_{\rm mod} = \hat{C}_{n_1 m_1}\cdots\hat{C}_{n_N m_N} = (-\mathrm{i})^N\delta_{n_1 m_1}\cdots\delta_{n_N m_N} + O(\text{loop})$$

Hang, HJH, arXiv:2106.04568, 2207.11214
Hang, Zhao, HJH, Qiu, arXiv:2406.12713

Generalized Power Counting Rule for KK Theories

- **Generalized Energy Power Counting for KK Theories:**

$$D_E = 2\mathcal{E}_{h_L} + (2L+2) + \sum_j \mathcal{V}_j\left(d_j - 2 + \tfrac{1}{2}f_j\right).$$

- **E-Power Counting of helicity-0 KK Graviton/Goldstone Amplitudes:**

$$D_E(Nh_n^L) = 2(N+L+1), \qquad D_E(N\phi_n) = 2(L+1),$$
$$\Longrightarrow \quad D_E(Nh_n^L) - D_E(N\phi_n) = 2N,$$

1

- **E-Power Counting of helicity-1 KK Graviton/Goldstone Amplitudes:**

$$D_E(Nh_n^{\pm1}) = N+2(L+1), \qquad D_E(N\mathcal{V}_n^{\pm1}) = 2(L+1),$$
$$\Longrightarrow \quad D_E(Nh_n^{\pm1}) - D_E(N\mathcal{V}_n^{\pm1}) = N.$$

- **E-Power Counting of KK Gauge/Goldstone boson Amplitudes:**

$$D_E(NA_L^{an}) = 4, \qquad D_E(NA_5^{an}) = 4 - N - \bar{V}_3^{\min}$$
$$\Longrightarrow \quad D_E(NA_L^{an}) - D_E(NA_5^{an}) = N + \bar{V}_3^{\min}$$

Hang, HJH, arXiv:2106.04568, 2207.11214

E-Cancellations of KK Graviton Amplitude via GRET

- **Gravitational Equivalence Theorem (GET) identity:**

$$\mathcal{M}[h^L_{n_1}(k_1),\cdots,h^L_{n_N}(k_N),\Phi] = \mathcal{M}[\phi_{n_1}(k_1),\cdots,\phi_{n_N}(k_N),\Phi] + \sum_{1\leqslant j\leqslant N}\mathcal{M}[\{\widetilde{\Delta}_{n_j},\phi_n\},\Phi],$$

- **Energy Power Counting:** $D_E[Nh^L_n] = 2(N+1)+2L\,,$

$D_E[N\phi_n] = 2+2L\,.$ $\qquad$ $D_E[N\tilde{v}_n] = 2+2L\,,$

- **We deduce h_L Amplitude has Large E-Cancellations:**

→ $D_E[Nh^L_n] - D_E[N\phi_n] = 2N$

- **For N = 4 scattering at tree level, GET proves Large E-cancellation:**

→ **$E^{10} \to E^2$ (by 10 - 2 = 8 powers)**

Hang, HJH, arXiv:2106.04568, 2207.11214

KK Gauge Amplitudes & E-Cancellations at 3-Point

- **3pt Longitudinal KK gauge boson amplitude ($A_L A_L A_T$) and KK Goldstone amplitude ($A_5 A_5 A_T$):**

$$\mathcal{T}_0[A_L^{an_1} A_L^{bn_2} A_\pm^{cn_3}] = -\mathrm{i} g f^{abc} (\epsilon_3 \cdot p_1) \frac{(M_{n_3}^2 - M_{n_1}^2 - M_{n_2}^2)}{M_{n_1} M_{n_2}} a_{n_1 n_2 n_3},$$

$$\widetilde{\mathcal{T}}_0[A_5^{an_1} A_5^{bn_2} A_\pm^{cn_3}] = -\mathrm{i} 2 g f^{abc} (\epsilon_3 \cdot p_1) \tilde{a}_{n_1 n_2 n_3}$$

- **The most fundamental GAET:**

$$\longrightarrow \quad \mathcal{T}_0[A_L^{an_1} A_L^{bn_2} A_\pm^{cn_3}] = -\widetilde{\mathcal{T}}_0[A_5^{an_1} A_5^{bn_2} A_\pm^{cn_3}]$$

enforcing energy cancellation: **$E^3 \rightarrow E^1$**

and it requires the coupling-mass condition:

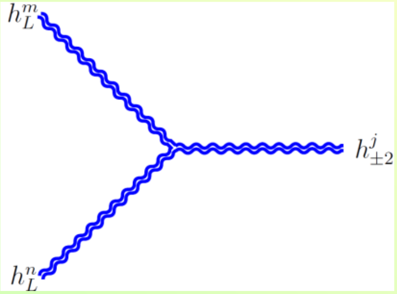
$$\longrightarrow \quad \left(M_{n_1}^2 + M_{n_2}^2 - M_{n_3}^2\right) a_{n_1 n_2 n_3} = 2 M_{n_1} M_{n_2} \tilde{a}_{n_1 n_2 n_3}$$

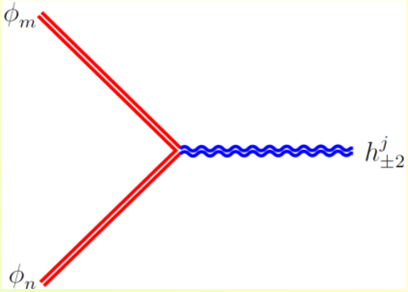
from which all N-point KK Amplitudes and GAET (N>3) can be derived !

Hang, Zhao, HJH, Qiu, JHEP(2025) [arXiv:2406.12713]

KK Graviton Amplitudes & E-Cancellations at 3-Point

- **3pt Longitudinal KK graviton amplitude ($h_L h_L h_T$) and KK Goldstone amplitude ($\phi_n \phi_m h_T$) :**





$$\mathcal{M}_0[h^L_{n_1} h^L_{n_2} h^{\pm2}_{n_3}] = \frac{\kappa(\epsilon_3 \cdot p_1)^2}{6\mathbb{M}^2_{n_1}\mathbb{M}^2_{n_2}}\left[2\mathbb{M}^2_{n_1}\mathbb{M}^2_{n_2} + \left(\mathbb{M}^2_{n_1} + \mathbb{M}^2_{n_2} - \mathbb{M}^2_{n_3}\right)^2\right]\alpha_{n_1 n_2 n_3},$$

$$\widetilde{\mathcal{M}}_0[\phi_{n_1}\phi_{n_2} h^{\pm2}_{n_3}] = \kappa(\epsilon_3 \cdot p_1)^2 \tilde{\beta}_{n_1 n_2 n_3},$$

- **E Cancellations for above 3-point KK Graviton Amplitude:** $E^6 \rightarrow E^2$

Hang, Zhao, HJH, Qiu, JHEP(2025) [2406.12713]

Gauge-Gravity Duality: Double-Copy
Massless vs Massive

Li, Hang, HJH, JHEP (2022), 2209.11191
Hang, Zhao, HJH, Qiu, JHEP (2025), 2406.12713
Hang, HJH, PRD (2022), 2106.04568, 2207.11214
Li, Hang, HJH, He, JHEP(2022), 2111.12042
Hang, HJH, Shen, JHEP(2022), 2110.05399
Research (2023), 2406.13671
H.X. Liu, Z.X. Yi, HJH, 2512.10870 [hep-th] (66pp)

$$(\mathrm{GR}) = (\mathrm{QCD})^2$$

$$(\mathrm{Gravity}) = (\mathrm{Gauge\ Theory})^2$$

$$(\text{引力}) = (\text{规范力})^2$$

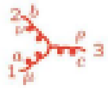
Double-Copy: KLT vs BCJ

- **for Amplitudes of Massless open/closed strings and for massless gauge/graviton bosons:**

1985: Kawai, Lewellen, Tye (KLT): "closed string amp=open-string amp^2"

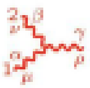
Field-theory limit:

Yang-Mills gauge theory:



$-gf^{abc}(\eta_{\mu\nu}(k_1-k_2)_\rho + \text{cyclic})$

Einstein gravity:



$i\kappa(\eta_{\mu\nu}(k_1-k_2)_\rho + \text{cyclic}) \times (\eta_{\alpha\beta}(k_1-k_2)_\gamma + \text{cyclic})$

"square" of Yang-Mills vertex.

gravity ↙ gauge theory color ordered ↙

$$M_4^{\text{tree}}(1,2,3,4) = -is_{12}A_4^{\text{tree}}(1,2,3,4)A_4^{\text{tree}}(1,2,4,3)$$

2008: Bern, Carrasco, Johansson (BCJ):

$$\mathscr{A}_4^{\text{tree}} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

$$\mathscr{A}_4^{\text{tree}}\Big|_{c_i\to n_i} \equiv -i\mathscr{M}_4^{\text{tree}} = \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

$$c_s + c_t + c_u = 0$$

$$n_s + n_t + n_u = 0$$

Massive Double-Copy vs Mass Spectral Condition

- For the massive KK graviton amplitude:

$$\mathcal{M}\left[h_{\sigma_1}^{\mathrm{n}_1} h_{\sigma_2}^{\mathrm{n}_2} h_{\sigma_3}^{\mathrm{n}_3} h_{\sigma_4}^{\mathrm{n}_4}\right] = -\frac{\kappa^2}{16} \sum_j \sum_{\lambda_k, \lambda'_k} \Big(\prod_{k=1}^{4} C_{\lambda_k \lambda'_k}^{\sigma_k} \Big) \frac{\mathcal{N}_j^{\mathrm{P}}(\lambda_k) \mathcal{N}_j^{\mathrm{P}}(\lambda'_k)}{s_j - M_{\mathrm{nn}_j}^2},$$

we impose the Generalized Gauge Transformation:

$$\mathcal{N}_j^{\mathrm{P}\,\prime} = \mathcal{N}_j^{\mathrm{P}} + \left(s_j - M_{\mathrm{nn}_j}^2\right) \times \Delta,$$

from which we deduce conditions:

$$\sum_j \mathcal{N}_j^{\mathrm{P}} = 0, \qquad \sum_j \left(s_j - M_{\mathrm{nn}_j}^2\right) = 0.$$

- This leads to the 4-point Mass Spectral Condition: → Nontrivial ! → Does Not always hold !

$$\sum_{i=1}^{4} M_{\mathrm{n}_i}^2 = M_{\mathrm{nn}_s}^2 + M_{\mathrm{nn}_t}^2 + M_{\mathrm{nn}_u}^2$$

Li, Hang, HJH, JHEP, 2209.11191

Massive Double-Copy vs Mass Spectral Condition

- Start from a general 4-point Mass Spectral Condition:

$$M_1^2 + M_2^2 + M_3^2 + M_4^2 = M_{12}^2 + M_{13}^2 + M_{14}^2$$

- We ask: How to solve it and What is the solution ??

- For the flat 5d Toroidal Compactification of S^1, we have:

$$n_1^2 + n_2^2 + n_3^2 + n_4^2 = (n_1 + n_2)^2 + (n_1 + n_3)^2 + (n_1 + n_4)^2$$

which leads to the condition:

$$n_1 + n_2 + n_3 + n_4 = 0$$

- This is just the KK number (5d momentum) Conservation!

- E.g., it does not hold for 5d Orbifold Compactification or 5d Warped Compactification ! (additional treatments needed.)

Li, Hang, HJH, JHEP, 2209.11191

Massive Double-Copy vs Mass Spectral Condition

- **Start from a general 4-point Mass Spectral Condition:**

$$M_1^2 + M_2^2 + M_3^2 + M_4^2 = M_{12}^2 + M_{13}^2 + M_{14}^2$$

- **We ask: How to solve it and What is the solution ???**

- **In fact, starting with 3 rather modest conditions:**

1). A massive theory contains at least 2 types of particles with Unequal masses.
2). There exists only a simple pole in each of (s, t, u) channels.
3). Each scattering amplitude should include the contributions from all 3 kinematic channels of (s, t, u).

we can prove:
→→ Toroidal Compactification of flat Extra dimensons is the Unique Solution !

Li, Hang, HJH, JHEP, 2209.11191

Massive Double-Copy vs Mass Spectral Condition

- **General 4-point Mass Spectral Condition:**

$$M_1^2 + M_2^2 + M_3^2 + M_4^2 = M_{12}^2 + M_{13}^2 + M_{14}^2 \qquad (1)$$

- **Proof of the Unique Solution:**

We identified the group structure underlying condition (1) is a product of Integer Groups Z^r (with rank r) in the Finitely Generated Abelian Group:

$$\mathcal{G} \cong \mathbb{Z}^r \oplus \mathbb{Z}_{p_1} \oplus \mathbb{Z}_{p_2} \oplus \cdots \oplus \mathbb{Z}_{p_s}, \qquad (r, s \in \mathbb{N})$$

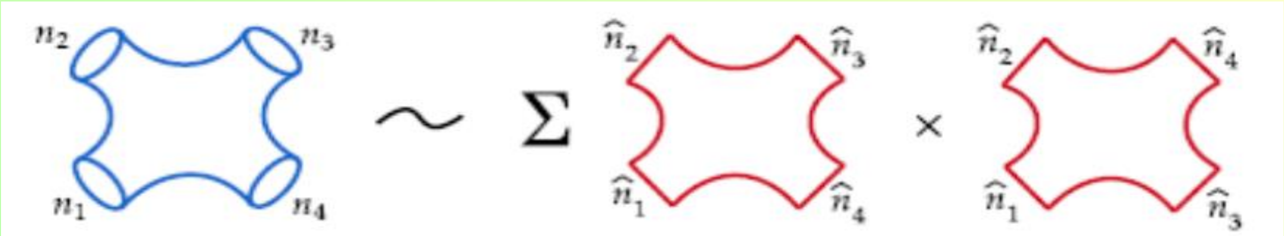
- **Then, we proved that the unique solution to Condition (1) is given by** $M_{\{k\}}^2 = k^2 M_{\{1\}}^2,$ **for the case r = 1 and by** $\mathrm{M}_{\mathrm{n}}^2 = \mathrm{n}\overline{\mathrm{M}}^2\mathrm{n}^T$ **for the case r>1 .**

- **By inspecting all the known consistent QFTs, we conclude that only the KK theories with δ (= r) extra dimensions under the toroidal compactification could have a mass spectrum behave exactly as above. These theories also hold the KK number conservation.**

Li, Hang, HJH, JHEP, 2209.11191

Gauge Forces vs Gravity Force

- **Massive Case:** **GR = (Gauge Theory)^2 ??**
- **What happens to massive Kaluza-Klein Theory ??**
- **First Principle Approach: Using KK bosonic string theory, we derived Massive KLT Relations between product of KK Open String Amplitudes and KK Closed String Amplitude. Taking Field Theory Limit, we derived Massive KLT Relations between product of KK Gauge Boson Amplitudes KK Graviton Amplitude:**



$$\mathcal{M}[1^n_L 2^n_L 3^n_L 4^n_L] = \frac{\kappa^2}{32} \sum_{\{a_j,b_j\}} \hat{\varrho}_{ab} \Big\{ (s-4M_n^2)\, \mathcal{T}_{a_j}[1^{+n}2^{+n}3^{-n}4^{-n}]\, \mathcal{T}_{b_j}[1^{+n}2^{+n}4^{-n}3^{-n}] + s\, \mathcal{T}_{a_j}[1^{+n}2^{-n}3^{+n}4^{-n}]\, \mathcal{T}_{b_j}[1^{+n}2^{-n}4^{+n}3^{-n}] + s\, \mathcal{T}_{a_j}[1^{+n}2^{-n}3^{-n}4^{+n}]\, \mathcal{T}_{b_j}[1^{+n}2^{-n}4^{-n}3^{+n}] \Big\}$$

Li, Hang, HJH, He, JHEP(2022), 2111.12042

Topological Mass Generation & Scattering Amplitudes
Topological Equivalence Theorem & Double-Copy
for Chern-Simons Gauge & Gravity Theories

H.X. Liu, Z.X. Yi, HJH, arXiv:2512.10870 [hep-th] (66 pp)
Hang, HJH, Shen JHEP(2022), 2110.05399
Research (2023), 2406.13671

3d Topological Massive Gauge & Gravity Theories

- **3d Spacetime has distinctive features characterized by Gauge & Gravitational Chern-Simons terms, which generate topological masses for gauge bosons and gravitons, realize fractional statistics and predict the existence of anyon-like quasi-particles.**

- **Chern-Simons (CS) terms are topological invariants in mathematics and serve as the theoretical key ingredients of a wide range of applications in modern physics, including fractional quantum Hall effect, models of high-temperature superconductivity and strongly correlated systems, & topological quantum computing.**

- **Studying structure of scattering amplitudes of massive gauge bosons and gravitons in the Chern-Simons theories provides an important means for understanding the mechanism of Topological Mass Generations and for realizing Gauge-Gravity duality connection: (Gravity) = $(\text{Gauge})^2$**

3d Topological Massive Gauge & Gravity Theories

- **3d Chern-Simons (CS) Topological Massive Gauge Theories:**

$$\mathcal{L}_{\mathrm{TMQED}} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\widetilde{m}\,\varepsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho\,,$$

$$\mathcal{L}_{\mathrm{TMYM}} = -\frac{1}{2}\mathrm{tr}\mathbf{F}_{\mu\nu}^2 + \widetilde{m}\,\varepsilon^{\mu\nu\rho}\mathrm{tr}\left(\mathbf{A}_\mu\partial_\nu\mathbf{A}_\rho - \frac{\mathrm{i}2g}{3}\mathbf{A}_\mu\mathbf{A}_\nu\mathbf{A}_\rho\right),$$

- **CS mass is geometrized and is related to CS Level:**

$$n = 4\pi\widetilde{m}/g^2 \in \mathbb{Z}$$

- **3d CS Topological Massive Gravity Theories:**

E^3

$$S_{\mathrm{TMG}} = -\frac{2}{\kappa^2}\int \mathrm{d}^3x\left[\sqrt{-g}R - \frac{1}{2\widetilde{m}}\varepsilon^{\mu\nu\rho}\Gamma^\alpha{}_{\rho\beta}\left(\partial_\mu\Gamma^\beta{}_{\alpha\nu} + \frac{2}{3}\Gamma^\beta{}_{\mu\gamma}\Gamma^\gamma{}_{\nu\alpha}\right)\right]$$

E^2

Deser, Jackiw, Templeton, Ann. Phys. (1982), PRL (1982)

Topological Mass Generation

- **Conversion of physical d.o.f between m = 0 and m ≠ 0 :**

$$A_{\rm P}^a = \frac{1}{\sqrt{2}}(A_{\rm T}^a + A_{\rm L}^a) \quad \longrightarrow \quad A_{\rm T}^a$$

- **2 orthogonal unphysical states:**

$$\begin{aligned} A_{\rm X}^a &= \epsilon_{\rm X}^\mu A_\mu^a = \frac{1}{\sqrt{2}}(A_{\rm T}^a - A_{\rm L}^a)\,, \\ A_{\rm S}^a &= \epsilon_{\rm S}^\mu A_\mu^a\,, \end{aligned}$$

- **The physical d.o.f is conserved under m → 0 limit:**

1 = 1

- **Then, we ask:**

$$\mathcal{T}[A_{\rm P}^{a_1},\cdots,A_{\rm P}^{a_N},\Phi] \overset{?}{=} \mathcal{T}[\tilde{A}_{\rm T}^{a_1},\cdots,\tilde{A}_{\rm T}^{a_N},\Phi]$$

Hang, HJH, Shen, JHEP, arXiv:2110.05399

Topological Mass Generation: TET

- **Indeed we can derive a new identity to connect the Scattering Amplitudes:**

$$\mathcal{T}[A_{\mathrm{P}}^{a_1}, \cdots, A_{\mathrm{P}}^{a_N}, \Phi] = \mathcal{T}[\tilde{A}_{\mathrm{T}}^{a_1}, \cdots, \tilde{A}_{\mathrm{T}}^{a_N}, \Phi] + \mathcal{T}_v \,,$$

$$\mathcal{T}_v = \sum_{j=1}^{N} \mathcal{T}[\tilde{v}^{a_1}, \cdots, \tilde{v}^{a_j}, \tilde{A}_{\mathrm{T}}^{a_{j+1}}, \cdots, \tilde{A}_{\mathrm{T}}^{a_N}, \Phi] \,,$$

- **Under high energy expansion, we derive, at S-matrix level,**
  Topological Equivalence Theorem (TET):

$$\mathcal{T}[A_{\mathrm{P}}^{a_1}, \cdots, A_{\mathrm{P}}^{a_N}, \Phi] = \mathcal{T}[\tilde{A}_{\mathrm{T}}^{a_1}, \cdots, \tilde{A}_{\mathrm{T}}^{a_N}, \Phi] + \mathcal{O}\left(\frac{m}{E}\right),$$

Hang, HJH, Shen, JHEP, arXiv:2110.05399

Topological Massive Gravity from Double-Copy

Hang, HJH, Shen, JHEP, 2110.05399

- **E Cancellations: $E^4 \to E^1$**

Amplitude	$\times \bar{s}_0^2$	$\times \bar{s}_0^{3/2}$	$\times \bar{s}_0$
$\mathcal{M}_s$	$-\frac{99+28c_{2\theta}+c_{4\theta}}{1-c_{2\theta}}$	$-\mathrm{i}\,14(15c_\theta+c_{3\theta})\csc\theta$	$-\frac{2(75+326c_{2\theta}+47c_{4\theta})}{1-c_{2\theta}}$
$\mathcal{M}_t$	$\frac{99+28c_{2\theta}+c_{4\theta}}{4(1-c_\theta)}$	$\mathrm{i}(102+105c_\theta+70c_{2\theta}+7c_{3\theta}+4c_{4\theta})\csc\theta$	$\frac{75-107c_\theta+326c_{2\theta}+268c_{3\theta}+47c_{4\theta}+31c_{5\theta}}{1-c_{2\theta}}$
$\mathcal{M}_u$	$\frac{99+28c_{2\theta}+c_{4\theta}}{4(1+c_\theta)}$	$\mathrm{i}(-102+105c_\theta-70c_{2\theta}+7c_{3\theta}-4c_{4\theta})\csc\theta$	$\frac{75+107c_\theta+326c_{2\theta}-268c_{3\theta}+47c_{4\theta}-31c_{5\theta}}{1-c_{2\theta}}$
Sum	0	0	0

- **Under high energy expansion:**

$$\mathcal{M}_0[4h_{\mathrm{P}}] = -\frac{\mathrm{i}\kappa^2}{2048}\, m\, s_0^{\frac{1}{2}}(494c_\theta+19c_{3\theta}-c_{5\theta})\csc^3\theta$$

- **Energy Cancellations for 4 graviton scattering amplitudes:**

$E^4 \to E^0$ → $\mathcal{O}(E^{12}) \longrightarrow \mathcal{O}(E^1)\,, \qquad (\text{for } \mathcal{E}_{h_\mathrm{P}}=4 \text{ in 3d TMG})$

Topological Mass Generation: Covariant TMG

- **Introduce dilaton field via conformal factor:**

$$g_{\mu\nu} = \bar{g}_{\mu\nu}\, e^{-\kappa\phi},$$

$$\mathcal{L}_{\rm TMG}^{\phi} = \frac{-1}{\kappa^2}\left\{\sqrt{-\bar{g}}\; e^{-\kappa\phi/2}\left[R+\frac{\kappa^2}{2}\bar{g}_{\mu\nu}\partial^\mu\phi\,\partial^\nu\phi\right]-\frac{\varepsilon^{\mu\nu\rho}}{2\tilde{m}}\Gamma^{\alpha}{}_{\rho\beta}\left(\partial_\mu\Gamma^{\beta}{}_{\alpha\nu}+\frac{2}{3}\Gamma^{\beta}{}_{\mu\gamma}\Gamma^{\gamma}{}_{\nu\alpha}\right)\right\}$$

- **Construct Gauge-fixing Terms:**

$$\mathcal{L}_{\rm TMG}^{\rm GF1} = \frac{1}{2\xi}\left(\mathcal{F}_{\rm GF1}^{\mu}\right)^2, \qquad \mathcal{F}_{\rm GF1}^{\mu} = \partial_\nu\bar{h}^{\mu\nu}-\frac{1}{2}\partial^\mu(\bar{h}-\xi\phi),$$

$$\mathcal{L}_{\rm TMG}^{\rm GF2} = \frac{1}{2\zeta}\left(\mathcal{F}_{\rm GF2}^{\mu}\right)^2, \qquad \mathcal{F}_{\rm GF2}^{\mu} = \frac{1}{2}\partial^\mu(\bar{h}-\zeta\phi),$$

- **Landau Gauge Propagators:**

$$\mathcal{D}^{h}_{\mu\nu\alpha\beta}(p) = -\frac{m p^\rho}{2p^2(p^2+m^2)}\left(\varepsilon_{\rho\mu\alpha}\bar{\eta}_{\nu\beta}+\varepsilon_{\rho\mu\beta}\bar{\eta}_{\nu\alpha}+\varepsilon_{\rho\nu\alpha}\bar{\eta}_{\mu\beta}+\varepsilon_{\rho\nu\beta}\bar{\eta}_{\mu\alpha}\right)$$
$$+\frac{\mathrm{i}m^2}{p^2(p^2+m^2)}\left(\bar{\eta}_{\mu\alpha}\bar{\eta}_{\nu\beta}+\bar{\eta}_{\mu\beta}\bar{\eta}_{\nu\alpha}-\bar{\eta}_{\mu\nu}\bar{\eta}_{\alpha\beta}\right),$$
$$D^{\phi}(p) = -\frac{\mathrm{i}}{p^2},$$

→ $1/p^3$

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Topological Mass Generation: Covariant TMG

- **Landau Gauge Propagators ($\zeta = 0$ and $\xi = 0$):**

$$\mathcal{D}^{h}_{\mu\nu\alpha\beta}(p) = -\frac{m p^{\rho}}{2p^2(p^2+m^2)}\left(\varepsilon_{\rho\mu\alpha}\bar{\eta}_{\nu\beta}+\varepsilon_{\rho\mu\beta}\bar{\eta}_{\nu\alpha}+\varepsilon_{\rho\nu\alpha}\bar{\eta}_{\mu\beta}+\varepsilon_{\rho\nu\beta}\bar{\eta}_{\mu\alpha}\right) + \frac{\mathrm{i}m^2}{p^2(p^2+m^2)}\left(\bar{\eta}_{\mu\alpha}\bar{\eta}_{\nu\beta}+\bar{\eta}_{\mu\beta}\bar{\eta}_{\nu\alpha}-\bar{\eta}_{\mu\nu}\bar{\eta}_{\alpha\beta}\right),$$

$$D^{\phi}(p) = -\frac{\mathrm{i}}{p^2},$$

1/p³

- **Unitary Gauge Propagator ($\zeta = \infty$ and $\xi = 0$):**

1/p²

$$D^{h(\mathrm{U})}_{\mu\nu\alpha\beta}(p) = \frac{-\mathrm{i}\left(\bar{\eta}_{\mu\alpha}\bar{\eta}_{\nu\beta}+\bar{\eta}_{\mu\beta}\bar{\eta}_{\nu\alpha}-\bar{\eta}_{\mu\nu}\bar{\eta}_{\alpha\beta}\right)}{p^2+m^2} + \frac{\mathrm{i}\xi}{p^4}\left(\eta_{\mu\alpha}p_{\nu}p_{\beta}+\eta_{\mu\beta}p_{\nu}p_{\alpha}+\eta_{\nu\alpha}p_{\mu}p_{\beta}+\eta_{\nu\beta}p_{\mu}p_{\alpha}\right) - \frac{m p^{\rho}}{2p^2(p^2+m^2)}\left(\varepsilon_{\rho\mu\alpha}\bar{\eta}_{\nu\beta}+\varepsilon_{\rho\mu\beta}\bar{\eta}_{\nu\alpha}+\varepsilon_{\rho\nu\alpha}\bar{\eta}_{\mu\beta}+\varepsilon_{\rho\nu\beta}\bar{\eta}_{\mu\alpha}\right) + \frac{\mathrm{i}}{p^2}\left(\bar{\eta}_{\mu\alpha}\bar{\eta}_{\nu\beta}+\bar{\eta}_{\mu\beta}\bar{\eta}_{\nu\alpha}-2\bar{\eta}_{\mu\nu}\bar{\eta}_{\alpha\beta}\right) - \frac{\mathrm{i}2}{p^4}\left(\eta_{\mu\nu}p_{\alpha}p_{\beta}+\eta_{\alpha\beta}p_{\mu}p_{\nu}\right),$$

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Topological Gravity ET in Covariant TMG

- **Topological Gravity Equivalence Theorem (TGRET) :**

$$\mathcal{M}[h_{\mathrm{P}}(p_1),\cdots,h_{\mathrm{P}}(p_N),\Phi] = \left(\tfrac{1}{2}\right)^N C_{\mathrm{mod}}\mathcal{M}[\phi(p_1),\cdots,\phi(p_N),\Phi] + O\left(\frac{m}{E}\right),$$

- **In comparison with TGAET:**

$$\mathcal{T}[A_{\mathrm{P}}^{a_1}p_1),\cdots,A_{\mathrm{P}}^{a_N}(p_N),\Phi] = \left(\tfrac{1}{\sqrt{2}}\right)^N \mathcal{T}[A_{\mathrm{T}}^{a_1},\cdots,A_{\mathrm{T}}^{a_N},\Phi] + O\left(\frac{m}{E}\right)$$

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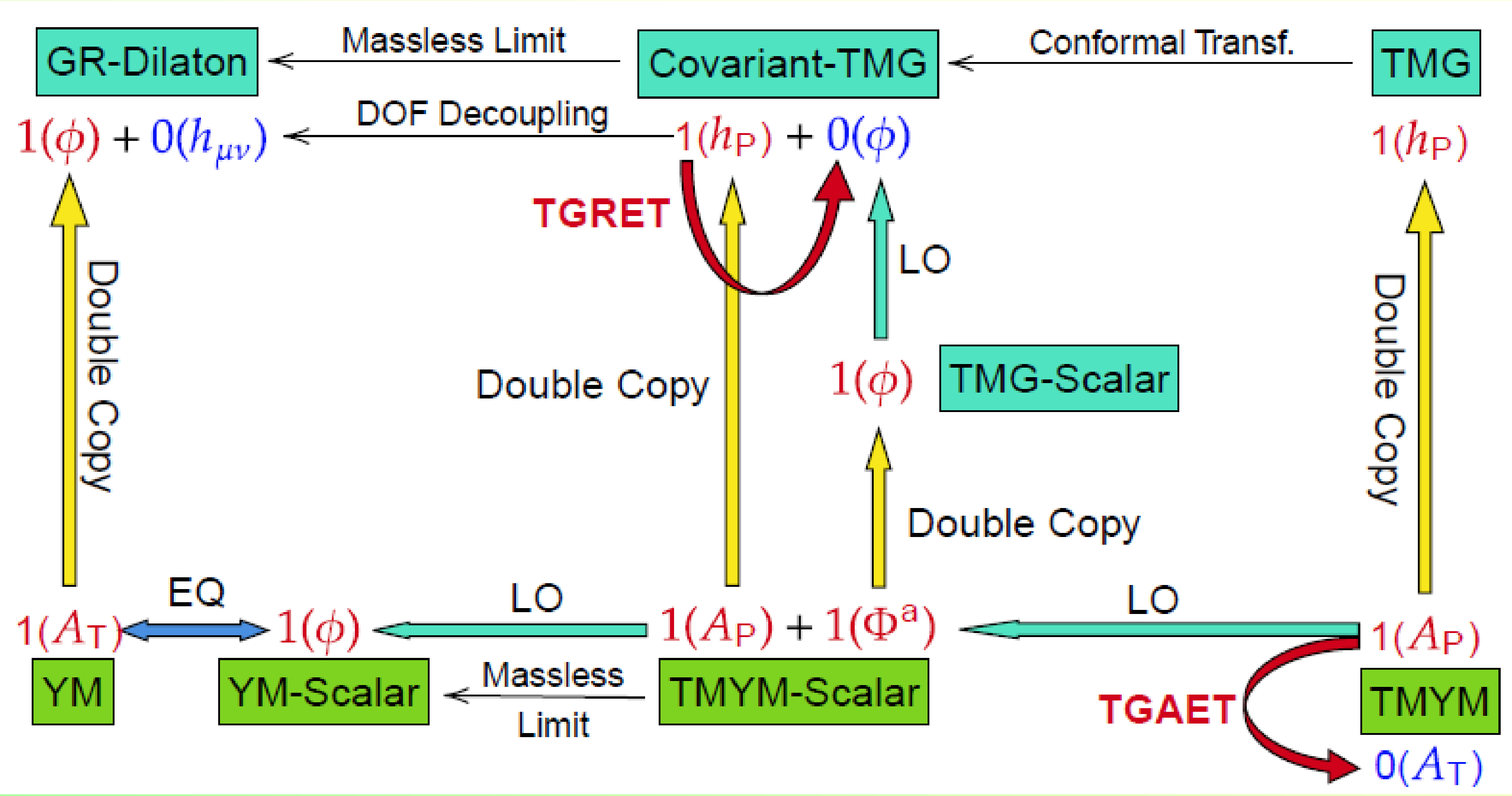
Topological Mass Generation: Summary



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Generalized Power Counting for 3d Chern-Simons

- **Generalized Energy Power Counting for 3d CS physical gauge boson scattering amplitudes:**

$$D_E = (\mathcal{E}_{A_p} - \mathcal{E}_v) + (4 - \mathcal{E} - \overline{\mathcal{V}}_3) - L\,,$$

- **Large E Cancellations in N physical A_p amplitudes by TET:**

$$\Delta D_E = D_E[NA^a_P] - D_E[NA^a_T] = N$$

- **E-Power Counting of N physical Graviton Amplitudes:**

$$D_E[Nh_P] = 2N+2+(\mathcal{V}-I_h+L) = 2N+3$$

$$D_E^U[Nh_P] = (2N+2)+\mathcal{V}+L = (2N+3)+I_h$$

$$D_{E(0)}[N\phi] = 3-n = 3-\tfrac{1}{2}N\,, \qquad (N=2n),$$

$$D_{E(0)}[N\phi] = 2-n = \tfrac{1}{2}(5-N)\,, \qquad (N=2n+1),$$

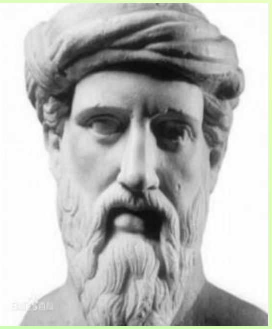
N=4 Scattering Amplitudes:

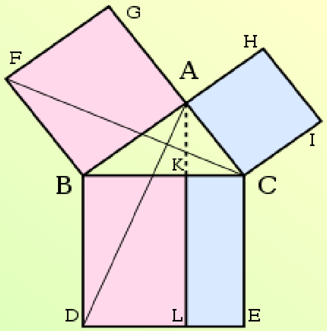
E^{12} → E^1 (Unitary Gauge)
E^{11} → E^1 (Landau Gauge)

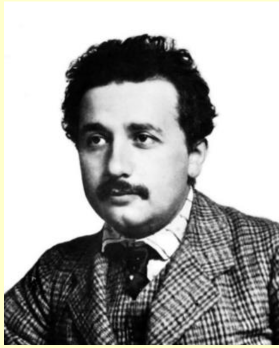
$$D_{E(0)}[Nh_P] - D_{E(0)}[N\phi] = \begin{cases} \tfrac{5}{2}N, & (\text{for } N=\text{even}), \\ \tfrac{1}{2}(5N+1), & (\text{for } N=\text{odd}), \end{cases}$$

$$D^U_{E(0)}[Nh_P] - D_{E(0)}[N\phi] = \begin{cases} \tfrac{7}{2}N-3, & (\text{for } N=\text{even}), \\ \tfrac{1}{2}(7N-5), & (\text{for } N=\text{odd}). \end{cases}$$

Deep Relations: Importance of Equality & Square Law







- [EG] Pythagoras Theorem: $a^2 + b^2 = c^2$ $\rightarrow$ Fermat Last Theorem:

  $a^n + b^n \neq c^n$ $(n>2)$
- [SR] Mass-Energy Equivalence: $E = Mc^2$ and $P_\mu^{\,2} = M^2c^2$
- [GR] Inertial Mass vs Gravitational-Mass Equivalence: $M_G = M_I$ $\rightarrow$ $g = a$
- Gravity Force from Gauge Force: (valid for $M = 0$ and $M \neq 0$)

$$(\text{Gravity}) = (\text{Gauge Force})^2$$

$M = 0$
(Conventional works)

$M \neq 0$
(This Talk)


→ Much more can be explored !
Thank You